

Modeling, Design, and Control of Flexible Manipulator Arms: Status and Trends

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ABSTRACT

The desire for higher performance manipulators has lead to dynamic behavior in which the flexibility is an essential aspect. This paper first examines the mathematical representations commonly used in modeling flexible arms and arms with flexible drives. Then design considerations directly arising from the flexible nature of the arm are discussed. Finally, controls of joints for general and tip motion are discussed.

1. MODELING FLEXIBLE ARMS

Models are used for simulation, analysis, and synthesis. In robotics, models may be used directly in the control algorithm with the computed torque technique. We will first look at the representation of the flexible behavior. Then, the incorporation of this flexible behavior into an overall arm model will be considered.

Modeling the Flexible Behavior

Examination of the energy storage characteristics of a component is helpful in assessing the modeling requirements of a system. Rigid arms store kinetic energy by virtue of their moving inertia and store potential energy by virtue of their position in the gravitational field. The flexible arm also stores potential energy by virtue of the deflection of its links, joints, or drives. Joints have concentrated compliance which is well modeled as a pure spring storing only potential energy. Drive components such as shafts or belts may appear distributed but store little kinetic energy due to their low inertia, and a lumped parameter spring model succeeds well for them also. Links are subject to torsion, bending, and compression. Torsion of a link stores potential energy but little kinetic energy due to low mass moment of inertia about the longitudinal axis of the beam and is thus well represented as a massless spring. Compression stores little potential energy due to the high compressional stiffness and dynamics along this axis is well described by a rigid mass. Links subject to bending store potential energy by virtue of their deflection as well as kinetic energy by virtue of their deflection rates and a good model must include this distributed nature. Partial differential equations result from an analysis of this type of problem, with time and one independent spatial variable usually adequate to represent the dynamic solution of the generally slender

links. The dynamics of the link itself may be represented by the Bernoulli-Euler beam equation:

$$EI \frac{\partial^4 y}{\partial x^4} + \mu \frac{\partial^2 y}{\partial t^2} = f(x,t) \quad (1)$$

which ignores shearing of the beam and the mass moment of inertia of a differential element along the length. The Timoshenko beam equation includes these two effects and should be used if the beam is short relative to its diameter. Such "stubby" links are likely to be essentially rigid anyway. The Bernoulli-Euler beam may have a varying cross section, but analytical solutions are not available except for the simplest variations. Thus the partial differential equation is useful to accurately model only very simple real links, or to study the general nature of the problem. This is not a great limitation since most time domain analysis is performed on a finite dimensional approximation of the distributed parameter system anyway.

Other assumptions are usually made when the partial differential equation is employed. While body forces may be included on the right side of (1), these forces due to translation and rotation of the link are quite complex and are only represented in simple cases not representative of telerobotics, such as a constant spinning satellite with an antenna. As discussed below, PDE's with simple boundary conditions are also used to obtain a set of basis functions. These spatially discretized equations may include body forces in at least an approximate way.

If we treat time as a continuous variable, an ordinary differential equation can be obtained by representing the beam shape as an infinite sum over a set of basis functions, each multiplied by its own time varying amplitude. Suitable approximations result from discarding all but a finite number of these functions as in (2). For arms with only rotational joints this procedure is quite natural.

$$y(x,t) = \sum_{i=1}^m a_i(t) \phi_i(t) \quad (2)$$

Prismatic joints on flexible links are another problem. As a flexible link moves in and out of a fixed prismatic joint its length effectively changes. Physically viewed, the energy stored in the link deflection must be transferred to another part of the beam. This is easily illustrated by a vibrating hack saw blade retracted over the edge of a table. The peak strain in the blade during the vibration must increase to store the same energy in a shorter blade and the frequency of the blade increases due to its higher natural frequency. Real manipulators will have real joints which are not ideally constrained, and must be examined for the degree to which they are able to constrain the translation, rotation and curvature of the link. These joints will dissipate vibrational energy as well, in fact this may be their most desirable feature. Research on modeling prismatic jointed flexible manipulators is limited.[1] Variation of the mode shapes is one approach to the problem, but it will not be discussed in detail in the remainder of the paper.

In spite of the long standing use of a truncated series of shapes to represent the kinematics of a flexible beam, no unimpeachable rule has evolved on the selection of these shapes to obtain the needed combination of model simplicity and accuracy. Popular choices are

1. Simple polynomials
2. Eigenfunction solutions to simple eigenvalue problems
3. Eigenvector solutions to finite element problems
4. Modal test results on the actual components.

Qualitatively speaking, the best results seem to come from shapes which allow the natural shape of the link when in the total system to be accurately described. Thus an "augmented body" which has the mass and/or inertia of the other links represented as a rigid appendage on the end of a given flexible link can be used to get the shape functions for that given link. This has given rise to endless tinkering with the link boundary conditions in an attempt to find the "perfect approximation." Some of these are described by Craig[2]. For manipulator arms this incorporates a wide range of effects varying with

1. Payload
2. Contact with the environment
3. Joint position
4. Accelerations due to joint motion
5. Feedback control law and gains.

With so much uncertainty we must accept either a) models of high order, b) models of low accuracy, or c) models not appropriate for the full range of operation. In particular, decisions made on the basis of these models should recognize their approximate and specialized nature.

The choice of mode shapes modifies the "rigid" motion variables. Different choices have different advantages. A clamped boundary condition leads to a physically measureable joint variable and simpler coefficients of the joint torques.[3] Pinned-pinned boundary conditions lead to ease in specifying the location of the joint tip and have been used to advantage in computed torque control since the tip position is specified by one variable per link.[4] Others have used free-free boundary conditions and described the link c.g. with "rigid" motion variables.[5]

Large Motion Equations

Implicit in the above description of the arm flexible components is the assumption of small excursions from a nominal position. This assumption could be violated in several ways. Nonlinear material behavior would result from strains beyond the elastic limit, for example. Even with linear elastic material behavior, one end of a sufficiently long beam can rotate through multiple revolutions with respect to the other end in violation of the small

motion assumptions of the derivation. A straight beam of length L projects a distance L onto an axis x . When one end of the beam is deflected from that axis and the other end is kept tangent to the x axis, the projected distance is no longer L , but must be less. The length of the beam remains constant at L , assuming no axial extension. The shorter projection on x is ignored as a standard procedure, but has been shown by Ryan and Kane[6] to lead to obvious errors under reasonable conditions of rotation. Likewise, centrifugal stiffening, often ignored, creates terms that affect the system dynamics. While the conditions of rotation of their example is unlikely for space robots, the point at which these phenomena become important is not clearly known and a point for future research.

Given the distributed behavior of a flexible link and the lumped behavior of joints and drives, overall equations of motion can be derived by several methods. A finite dimensional model will be assumed here. Lagrange's equations and several variations thereon are readily applied. Kane has formulated equations in a manner of some distinction.[7] Newton's laws are less popular because of the distributed nature of the flexible links. Only the first, Lagrange's equations, are discussed at all here. Lagrange's equation for flexible links differ from rigid links because the flexible degrees of freedom appear principally as a sum rather than as a product due to the parallel contribution of each of the assumed shape functions. This is easily illustrated with the homogeneous 4×4 matrix formulation of the forward kinematics. For a rigid arm, the end point coordinate transformation is:

$$T = A_1 A_2 A_3 A_4 A_5 A_6 \quad (3)$$

where A_i is the transformation representing the joint and link. For the flexible arm the end point coordinate is

$$T = A'_1 E_1 A'_2 E_2 A'_3 E_3 A'_4 E_4 A'_5 E_5 A'_6 E_6 \quad (4)$$

$$E_i = H_i \sum_{j=1}^{m_i} \delta_{ij} M_{ij}$$

where H_i transforms along the length of the undeformed i -th beam and M_{ij} adds the effect of mode j . A'_i transforms for the joint only.

where the E_i 's incorporate the summation of the assumed "mode" shape transformations $E_{i,j}$ and L_i incorporates the undeformed transformation of the link length. When an integration over the length of the flexible beam is used to incorporate all the kinetic and potential energy contributions, the assumed shapes are integrated to yield, for example, "modal masses", "modal stiffnesses", and "modal input matrix elements". These become coefficients in the final dynamic equations and are one way the choice of shape influences the final result. The choice of shape also will affect the calculation of outputs from the model, such as end point position or strain in the links. If a finite element or experimentally generated shape is used, equivalent integrals are approximated from the nodal points in the model.

The coefficients described above involve cross products of the various shape functions. The choice of orthogonal shape functions eliminates a number of terms from the final equations of motion, since for orthogonal functions

$$\int_0^L \phi_i(x) \phi_j(x) dx = 0, i \neq j \quad (5)$$

The orthogonality condition is automatically obeyed by the eigenfunctions of components. If boundary conditions are chosen to represent, for example, the mass of an augmented body the orthogonality condition will incorporate terms outside the integral. The true net value of this simplification is not clear. Operations such as inversion of the inertia matrix and multiplication by the inverse eliminate the zero coefficients. In simulation the coefficients may be recalculated relatively infrequently. Some researchers support the use of polynomials because they are able to represent more general conditions on the flexible link. Even though a polynomial is not orthogonal it is simple to compute. Incorporating the orthogonality condition correctly complicates the derivation procedure with the hope of ultimately reducing the complexity of the final equations. Since the equations are so complex already for practical cases, either symbolically generated equations or general multi body codes are the only practical way to generate reliable equations. A final judgment on the use of orthogonal shapes should be made in the context of that implementation.

Lagrange's equations applied by brute force to the appropriate energy functions will generate a complex dynamic model. Simplification can be achieved by simplification of the resulting complex equations can be pursued as described by Book[3]. Remarkable success has recently been achieved by prior simplification, using the specific form of the equations and the relation between the coefficients ultimately sought for the equations of motion.[8][9] Kinetic energy, for example, can be written as the integral over the link, but also as a quadratic product with the rate variables with the mass matrix.

$$KE = \dot{x}^T M \dot{x} / 2 \quad (6)$$

The nonlinear dynamic terms can be related to the changes of the mass matrix. Various relations like this have been used to represent rigid arm dynamics[10] and their analogies are now being found for flexible arms.

Non-serial arms

Very few of the thousands of manipulators constructed in the world qualify exactly as serial link manipulators. Only a pure direct drive arm can meet the qualification since all speed reductions involve parallel structural and drive elements. Incorporating parallel flexible links is difficult but manageable.[11] Differential equations can be formulated for each parallel path along the structure up to a point where they must connect. Algebraic constraint equations prescribing the meeting of the parallel paths must accompany the differential equations and their constraint forces. Several numerical techniques are then available to jointly solve these two types of equations and eliminate the extra degrees of freedom from the differential equations. Among the numerical procedures relevant are Singular Value Decomposition (SVD), QR decomposition, LU partitioning and Gaussian elimination.[11] SVD is attractive for flexible manipulators because the reduced variables resulting are tangent to

the constraint surface, and errors in satisfying the constraint equation have less effect on the overall dynamic simulation.

Symbolic Derivation and Multi-Body Codes

Hand derivation of dynamic equations for multi-link flexible arms is not recommended for producing the final equations of motion. It does give a student of the subject a healthy appreciation for the complexity of the problem and perhaps ideas for simplifying the result. Two alternatives are symbolic derivation and the use of general purpose multi-body codes.

General purpose symbolic manipulation programs include MACSYMA, SMP, REDUCE, and MAPLE. These were originally developed for main frame and mini computers but similar programs are now available on work stations and even personal computers. Special purpose symbolic codes are available for rigid arms (SDExact and SDFast) and are under development for similar flexible systems[12]. The general purpose symbolic codes allow one to approach research on more complex configurations with confidence. They cannot be viewed as an automatic means to turn theory into simulation code. Problems of practical complexity can swamp even large memories with "intermediate expression swell," especially when an expression is expanded in preparation to simplification. Pathological cases continue to be found on these systems which give incorrect results without even warning the user. They are extremely complex programs in general, and some have evolved over many years leading to both good and bad characteristics. While not a panacea, they are an invaluable, almost essential to one who would develop equations of motion for a flexible arm.

Guidance on the use of general purpose manipulation programs for rigid[13] and flexible manipulators using Lagrange's equations[14] is available in the literature. The special nature of kinematic chains leading to symmetry and zero terms can be exploited well with symbolic manipulation as shown in recent research.[8,9] for both serial and non serial arms.

Multi-body simulation codes are capable of handling unconstrained and in some cases constrained flexible chains. Well known codes include DISCOS, CONTOPS, DADS, ADAMS and others.[15] These codes insert the specifics of the model in numerical form early in the equation generation process. The disadvantage is more computational burden at each simulation time step. The advantage is a very general formulation suitable for components connected in a tree topology by various joints. They generally accept assumed shape data directly from finite element analysis modal results. Simplified linear models can be generated for more intensive design tradeoff studies on control, for example. In our research with flexible arms we have begun to look to these codes for independent verification of the accuracy of symbolically generated models.

Model order reduction

Discretization of the partial differential equations reduces the order of the flexible arm model from infinity to n . The value of n and the flexible degrees of freedom to be included can be determined from modal cost analysis techniques[16] that are most relevant to structures without feedback controls or to structures with the feedback controls already included. Tsujisawa[17] has applied this analysis to planar motion of a large arm with parallel actuated second link and found that one or two modes per flexible link were adequate for

his case. Large space structures may require dozens or hundreds of modes. The difference is the relatively clean, simple nature of the structure that Tsujisawa examined and that arms in general exhibit. Modal cost analysis does not insure that some higher mode will not cause instability i.e. that the effects of observation spill over will be small. This is especially problematical when the passive damping of the structure is small. Alberts[18] has shown the pronounced effects on stability of enhancing the passive damping through surface treatments. No general guarantee that a model includes the right degrees of freedom exists. A high order "truth model" for ultimate verification is perhaps the only insurance short of experiments.

Frequency Domain Analysis

It should be mentioned that if PDE descriptions of flexible elements are accurate and large motion behavior is not of interest, A very attractive alternative is frequency domain analysis. Models can be composed of elements which are flexible or rigid, serial or parallel, with up to six axes of freedom. Serial connection of components is much more readily incorporated using the transfer matrix approach. With the transfer matrix approach facilitates creation of the model from a library of elements as described in Book.[19] The principal price paid is a restriction to linear behaviors. While the powerful time domain synthesis techniques are not directly applicable to the frequency domain model, iterative techniques for pole placement were used by Book and Majette which converted between frequency domain and simple time domain models. Readily available are frequency response, natural frequency, true mode shapes, and, via the inverse FFT time response. The models have been applied to the Space Shuttle Manipulator Arm and a complex payload with general spatial motion. Numerical accuracy limited the approach as Majette used it, even with a 60 bit word length. More robust numerical techniques have been used in analysis of spinning spacecraft.[20] This approach is perhaps under utilized in the dynamics and control community. The powerful finite element techniques have dominated and seem more relevant to "messy problems" with many appendages and parallel connections.

2. DESIGN OF FLEXIBLE ARMS

This section is about the design of arms which behave well even though they are flexible. Designing arms to be flexible is not of practical interest.

Material properties principally affect strength, stiffness and damping. High strength materials allow lighter cross sections, consequently more flexibility. For many high performance materials, strength and stiffness increase together. While stiffness determines the need for flexible arm control algorithms, damping determines the ease in implementing such a control algorithm. Composite materials typically have more damping than homogeneous metals. Cost and ease of working with metals is a strong incentive for alternative means of damping enhancement.

The constrained layer damping treatment is a very effective means of enhancing damping for flexural vibrations.[18] One can achieve an increase in damping by a factor of 10. The treatment consists of a thin visco-elastic layer placed on the beam's surface and covered by a very stiff constraining layer. Bending of the beam results in shearing of the visco-elastic material and consequent dissipation of energy. This dissipation can be maximized for a given wavelength

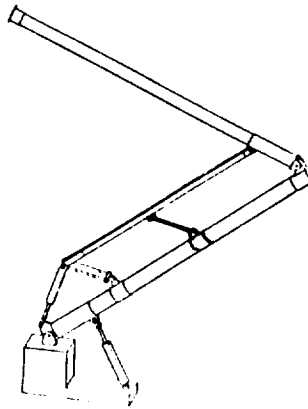


Figure 1: Robotic Arm Large and Flexible (RALF). Note multiple connections between parallel linkages.

of vibration by sectioning the constraining layer. The dissipation can be large for a wide range of wavelengths and their attendant frequencies, however. Torsional vibrations are not so readily coupled to the constrained layer treatment. A spiral wrap of a strip of constraining material has been examined by Dickerson[21] and was effective for this purpose. Vibrations with torsional compliance of a link depends on other links or the payload for inertia. These links would be at an angle to the link in torsion and hence undergo some flexure. This flexure could be effectively damped as mentioned before.

An active alternative to passive damping is active damping using piezo electric films or ceramics.[22] By closing the loop with local measurements of the strain, much the same effect of passive damping is obtained. One advantage of the active approach is the elimination of temperature sensitivity which is quite pronounced in the visco-elastic materials. Another advantage is a small amount of static deflection that can be obtained to control the shape of structures if that is important. The disadvantage is the complexity of an additional control loop and a high excitation voltage of several hundred volts.

Designs to minimize the flexibility of an arm are also important. Parallel link mechanisms are more rigid than a serial link mechanisms of equivalent weight such as the Stuart's platform. Unfortunately, the range of travel is typically smaller too. A parallel actuation link can have the dual benefits of placing motor mass near the base and providing a larger cross section area moment of inertia when the two parallel links are bent. Multiple connections between the parallel links such as shown in Fig. 0.1 work to further stiffen the pair by allowing both to support the load in bending. The added attraction of this arrangement is that the buckling load for the actuation link is increased.

Design should be interpreted broadly to include completely new concepts of arm operation. Additional degrees of freedom, e.g. a small arm, on the end of a long flexible arm are one way to change the nature of tradeoffs that must be made in arm design. The tradeoff between arm rigidity and low inertia can be couched in terms of gross and fine motion speeds. The extra degrees of freedom can be used to have light weight for gross motion of a large arm and high bandwidth rigid motion of a small arm it carries. These extra degrees of freedom can also be used to generate inertial forces that act to reduce vibration much as a dynamic vibration absorber would. They can also be used to

compensate for the relatively slow vibrations of a large arm, keeping the end point stationary. These three strategies are discussed in a companion paper, and will not be dealt with in detail here.

3. CONTROL FOR JOINTS OF FLEXIBLE ARMS

This section will discuss the limits for performance with rigid arm controls and the use of advanced control algorithms of various types. The control of only the existing joints will be discussed, not the addition of additional actuators specifically for controlling vibrations, such as proof masses, reaction wheels, or "smart materials." Such a control problem for robots is typically broken down into path planning, trajectory planning, and trajectory following. Little if any work on path planning specifically for flexible arm robots has been presented. The concentration will therefore be on the other two phases. For flexible robots control might also have the objective of vibration damping.

Trajectory Planning

Trajectory planning for the joint and the end point of a flexible arm are not equivalent problems as they are for the rigid arm. Three perspectives on the problem can be identified:

1. Generate an "optimal" trajectory
2. Control the arm's tip to follow a specified trajectory
3. Control the joints account for rigid link motion plus static deflection and suppress arm vibrations with feedback control.

Sangveraphunsiri[23] numerically determined the minimum time trajectory for a single link arm but recommended a near optimum based on rigid behavior and a feedback control near the final position due to the sensitivity of the true optimum to parameter variations. Meckl and Seering[24] solved a similar linearized problem in modal coordinates which yielded a simpler solution format applicable to the complete class of linear problems. Book and Cetinkunt[25] looked at trajectory optimization for rigid arms and extended it in an approximate way to flexible arms.

Singer and Seering[26] examined shaping techniques for trajectories in terms of their vibrational consequences. Work by Oosting and Dickerson[27] sought to choose tip trajectories to make the joint torques realizable without preview. A type of inverse dynamics model was used. Bayo[28] used a more elaborate model to obtain joint torque histories to track a Gaussian tip trajectory. Bayo's model was based on finite element and his original solution involved frequency domain techniques with substantial computational burden. He is extending his results to the two link case. These inverse dynamics approaches must simultaneously incorporate inverse kinematics, since the flexible degrees of freedom cannot be decoupled from the joint degrees of freedom. Asada also studied the inverse dynamics/kinematics problem and found that it was possible to produce a well behaved tip motion that resulted in unstable joint motion.[4] This was not observed by Bayo or Dickerson and may involve the choice of trajectories to be followed.

Several authors have planned the trajectory based on a static correction to the tip position based on rigid arm motion. Since this is not a dynamic correction it is easily implemented. Joint position adjustments assume instantaneous wind up of the arm "spring" predicted by acceleration forces. The method is shown to be very successful. It would seem that a good choice of trajectory to track is critical to the success of this method.

It should also be mentioned that if the tip position is not crucial except near the end of a motion, it is quite reasonable for the flexible arm to track joint motions based on rigid kinematics and use a final feedback control to produce acceptable vibration characteristics. When a flexible manipulator is used as a teleoperator, trajectories are at the disposal of the human operator, and can be at most minimally filtered to condition the reference signal.

Trajectory Tracking

As an arm is made lighter (more flexible) a point will be reached when it can no longer be considered rigid. Alternatively, if the arm servo bandwidth, ω_s is increased sufficiently, interaction of the dominant poles with the lowest ignored poles (in the rigid model) will eventually occur. For simple beams connected by rotary joints this lowest pole will be the first structural frequency with joints clamped, ω_c . The proximity of the closed loop bandwidth to ω_c gives one a valuable measure of the difficulty in achieving that bandwidth with simple rigid arm controllers. For example $\omega_s < \omega_c/2$ has been proposed as a practical limit for simple P.D. joint control.[29]

Imperfections in the system's behavior result in poor performance even for much stiffer systems. Coulomb friction in the joints, for example, will result in the link not back driving the actuator for small vibrations. These oscillations cannot be damped by the actuator motion, i.e. energy cannot be removed from the vibration. With greater frictional break away torque, larger amplitude vibrations will be allowed to exist with only structural damping slowly reducing their amplitude. With link strain included in the joint control, even low amplitude structural vibrations can be damped. For arms with speed reducers this is especially valuable, since some high ratio reducers are not back driveable under any circumstances. The strain feedback can be viewed either as a damping enhancement or as an inner torque control loop.

In order to account for a limited number of additional states in a flexible arm, a higher order model can be used in the control synthesis. Linear regulators or tracking controls optimized based on a quadratic performance index with a guaranteed margin of stability have been employed for end point[30] and strain measurements[31] for one link arms and also for multi link arms[32][8]. Measurements involving link flexure can introduce non-minimal phase behavior. This is most apparent with tip position measurements of a one link arm. The tip initially moves in the opposite direction of the applied torque. The linear transfer function of such a system has zeros in the right half plane. The poles of an output feedback controller will move toward these zeros as gains increase, leading to instabilities. Viewed from a state space perspective, some optimization techniques for linear systems effectively cancel unwanted zeros with poles, leading to instability when the cancellation is inexact. Other measurements are less vulnerable to non minimum phase difficulties. Strain gages at the base of a one link arm with motor inertia has no non minimum phase zeros, and yet can be used to observe all system states.[33] The price paid is

the lack of direct knowledge about end point position in the uncertain work environment. For higher modes it is desirable to make more measurements instead of having a high order observer, with the attendant computational requirements. These additional sensors can introduce the non minimum phase zeros and instability. Multi link arms introduce an even greater need for additional measurements. Understanding more thoroughly the role of non minimum phase dynamics in both the linear and nonlinear cases is a very challenging and potentially useful research area. How can one combine strain and tip position sensors to achieve a robust and accurate controller?

The system linearity assumed in standard LQR control design is highly questionable in the robotic applications. Rigid arm control has been able to circumvent this through various means, including computed torque techniques, linearizing controllers, nonlinear controls (e.g. variable structure control), and adaptive controls. When the number of degrees of freedom exceeds the number of actuators, as for flexible arms, this approach must be modified.

Adaptive control has also been applied to the rigid case. Application of rigid arm model reference adaptive control to a flexible arm can overcome the adverse nonlinear forces at high velocities, but cannot overcome the bandwidth limitations imposed by vibrational structural modes.[34] For a flexible MRAC to be designed using the stability methods applied to rigid arms, the "model matching" condition must be satisfied. The rigid arm development uses the equal numbers of actuators and degrees of freedom. The near linearity of the one link flexible case has allowed Siciliano, et.al.[35] to accomplish model matching to a reference model which, instead of decoupling the rigid degrees of freedom, is a linearized model of the flexible arm. This also provides values of the flexible states for tracking during the motion. Others have proposed indirect adaptive control approaches such as the estimation of the payload mass.[36]

An explicit means of incorporating model uncertainty and simplifying assumptions is provided by the bounded uncertainty approach. Robustness can be enhanced by adding to the linear control adaptation and a saturation term similar to variable structure controls. This technique has been used to derive a decentralized controller for a two joint flexible arm with very good success relative to both a rigid controller and to a pure linear flexible feedback controller.[8]

The complexity of the nonlinear, flexible problem has lead researchers to seek various ways to simplify the problem. Rigid-nonlinear approximations are usual. Flexible-linear approaches are also common. When limited to small motions and inconsequential nonlinear velocity forces this is straight forward. Since the elastic deflections are usually rather small, linearization along a specified motion path is also effective. It is not accurate to assume the gross motions only force the flexible motions, since the damping of the vibrations is increased by their influence on the moving joint. One well developed approach for separating rigid and flexible motions is by exploiting their time scale separation with Singular Perturbation Analysis. This has been studied for arms with compliant drives[37] and with flexible links[38]. If flexible frequencies are to retain the broad separation from the "rigid body" frequencies needed for the singular perturbation theory to hold, high performance light weight arms will be automatically excluded. An alternative to including all flexible degrees of freedom in the fast system is to place the lowest mode in the slow system with the rigid body modes. This enables the dominant dynamics to remain

in the slow system. It appears feasible for light arms with significant payloads. For example, as the payload of a beam gets bigger and heavier, its first bending frequency approaches zero. The second bending frequency approaches the first clamped-clamped bending frequency. On a percentage basis, the separation of the two modes increases as the payload increases.

Practical advantage seems to be gained when the decoupling of rigid and flexible motion combines a linearizing feedback control for the rigid motion and a linear control on the flexible subsystem linearized about the rigid motion. A static deflection correction to the specified joint motion can also be incorporated to place the tip closer to its specified trajectory.[39]

4. CONCLUSIONS

The consideration of flexibility in manipulator arms is in a rapid state of progress relative to a few years ago, but much remains to be done. Many new approaches in modeling, design and control are being explored. It is important that experimentation accompany the theoretical and simulation results to keep realism in the research. Even single link experiments are much better than no experiments. It is important to move into realistic 2 and 3 joint experiments where flexibility is representative of real applications or at least scaled to those applications. It is possible that earlier work on rigid arms will find more application when the role of flexibility is more fully understood.

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